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A Note on Thermodynamics of Black Holes in Lovelock Gravity

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Abstract

The Lovelock gravity consists of the dimensionally extended Euler densities. The geometry and horizon structure of black hole solutions could be quite complicated in this gravity, however, we find that some thermodynamic quantities of the black holes like the mass, Hawking temperature and entropy, have simple forms expressed in terms of horizon radius. The case with black hole horizon being a Ricci flat hypersurface is particularly simple. In that case the black holes are always thermodynamically stable with a positive heat capacity and their entropy still obeys the area formula, which is no longer valid for black holes with positive or negative constant curvature horizon hypersurface. In addition, for black holes in the gravity theory of Ricci scalar plus a $2n$-dimensional Euler density with a positive coefficient, thermodynamically stable small black holes always exist in $D = 2n+1$ dimensions, which are absent in the case without the Euler density term, while the thermodynamic properties of the black hole solutions with the Euler density term are qualitatively similar to those of black holes without the Euler density term as $D > 2n+1$.

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Over the past years gravity with higher derivative curvature terms has received a lot of attention, in particular, in the brane world scenario and black hole thermodynamics. In the former, the motivation is to study the corrections of these higher derivative terms to the Newton law of gravity on the brane, or to avoid the singularity in bulk by using these higher derivative terms. In the field of black hole thermodynamics, it is expected to gain some insights into quantum gravity since the thermodynamic property of black hole is essentially a quantum feature of gravity. On the other hand, it is possible through the thermodynamics of black holes with AdS asymptotic to study the thermodynamic properties and phase structure of a ceratin field theory because due to the AdS/CFT correspondence, some higher derivative curvature terms can be regarded as the corrections of large N expansion of dual field theory.

Among the higher derivative gravity theories, the so-called Lovelock gravity [1] is quite special, whose Lagrangian consists of the dimensionally extended Euler densities

$$\mathcal{L} = \sum_{n=0}^{m} c_n \mathcal{L}_n, \tag{1}$$

where c_n is an arbitrary constant and $\mathcal{L}_n$ is the Euler density of a $2n$-dimensional manifold:

$$\mathcal{L}_n = 2^{-n} \delta^{a_1 b_1 \cdots a_n b_n}_{c_1 d_1 \cdots c_n d_n} R^{c_1 d_1}{}_{a_1 b_1} \cdots R^{c_n d_n}{}_{a_n b_n}. \tag{2}$$

Here the generalized delta function is totally antisymmetric in both sets of indices. $\mathcal{L}_0$ is set to one, the constant c_0 is therefore proportional to the cosmological constant. $\mathcal{L}_1$ gives us the usual curvature scalar term. In order the Einstein's general relativity to be recovered in the low energy limit, the constant c_1 must be positive. Here, for simplicity, we just set the constant $c_1 = 1$. The $\mathcal{L}_2$ term is the Gauss-Bonnet one, which often appears in the recent literature. Except for the advantage that the equations of motion of the Lovelock gravity, as the case of the Einstein's general relativity, do not contain terms with more than second derivatives of metric, the Lovelock gravity has been shown to be free of ghost when expanding on a flat space, evading any problems with unitarity [2]. Here it should be stressed that although the Lagrangian (1) consists of some higher derivative curvature terms, the Lovelock gravity is not essentially a higher derivative gravity since its equations of motion do not contain terms with more than second derivatives of metric. Just due to this, the Lovelock gravity is free of ghost [3].

In the literature concerning on the Lovelock garvity, the extensively studied is the so-called Gauss-Bonnet gravity, whose Lagrangian is the sum of the curvature scalar term $\mathcal{L}_1$ and the Gauss-Bonnet term $\mathcal{L}_2$, the Euler density of a 4-dimensional manifold. Sometimes, a cosmological constant is added to the Lagrangian. In this theory, the static, spherically

symmetric black hole solution was found in Refs. [2, 4]. The black hole solutions with nontrivial horizon topology were studied in Ref. [5]. Refs. [6, 7] discussed some aspects of holography of the Gauss-Bonnet gravity. In particular, it is worth mentioning here that with a positive Gauss-Bonnet coefficient c_2, in spite of the asymptotic behavior (asymptotically (anti-)de Sitter [5, 8] or flat [9]) of black hole solutions, it is found that a locally stable small black hole always appears when the spacetime dimension $D = 5$, which is absent in the case without the Gauss-Bonnet term, while $D \geq 6$, the thermodynamic behavior of the Gauss-Bonnet black hole is qualitatively similar to the case without the Gauss-Bonnet term (see also related discussions in [10, 11]).

The Lagrangian (1) looks complicated. It is therefore a little surprise to know that the static, spherically symmetric solution can be found in the sense that the metric function is determined by solving for a real root of a polynomial equation [4]. Since the gravity (1) includes many arbitrary coefficients c_n, it is not an easy matter to extract physical information from the solution. In Refs. [12, 13] by choosing a special set of coefficients, the metric function can be expressed in a simple form. These solutions could be explained as spherically symmetric black hole solutions. Black hole solutions with nontrivial horizon topology in this gravity with those special coefficients have also been studied in Refs. [14, 15].

For the general case with arbitrary coefficients c_n, the static, spherically symmetric solution was found in Refs. [4, 9]

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_D^2, \tag{3}$$

where $d\Omega_D^2$ denotes the line element of an $(D-2)$-dimensional unit sphere and the metric function $f(r)$ is given by

$$f(r) = 1 - r^2 F(r). \tag{4}$$

$F(r)$ is determined by solving for real roots of the following mth-order polynomial equation

$$\sum_{n=0}^{m} \hat{c}_n F^n(r) = \frac{16\pi G M}{(D-2)\Omega_D r^{D-1}}. \tag{5}$$

Here G is the Newton constant in D dimensions, $\Omega_D = 2\pi^{(D-2)/2}/\Gamma[(D-2)/2]$ is the volume of an $(D-2)$-dimensional unit sphere, M is an integration constant, and the coefficient $\hat{c}_n$ is given by

$$\begin{aligned} &\hat{c}_0 = \frac{c_0}{(D-1)(D-2)}, \qquad \hat{c}_1 = 1, \\ &\hat{c}_n = c_n \Pi_{i=3}^{2m}(D-i) \qquad \text{for } n > 1. \end{aligned} \tag{6}$$

The asymptotic behavior and causal structure of the solution have been analyzed in detail by Myers and Simon in Ref. [9]. It is found that even when the cosmological constant c_0 vanishes, the solution could be asymptotically de Sitter, flat, or anti-de Sitter, which depends on the coefficients c_n,(in other words, it can be seen from (4) that the solution is asymptotically de Sitter, flat, or anti-de Sitter if $F(r)$ approaches to a positive constant, zero or a negative constant as $r \to \infty$, respectively) and that (black hole/cosmological) horizon structure is quite rich. But we do not repeat them here. Further it is easy to conclude that the integration constant M is the ADM mass when the solution is asymptotically flat, while it corresponds to the AD mass for the asymptotically (anti-)de Sitter case [16].

Nowadays it is well-known that in asymptotically anti-de Sitter space, the black hole horizon could be topologically nontrivial: the horizon can be a closed hypersurface with positive, zero, or negative constant curvature [17]. Such black holes are called topological black holes. Now we generalize the spherically symmetric solution (3) to more general case:

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Sigma_D^2, \tag{7}$$

where $d\Sigma_D$ denotes a line element of an $(D-2)$-dimensional hypersurface with constant scalar curvature $(D-2)(D-3)k$ and volume Σ_D. Here k is a constant. Without loss of generality, k can be set to ± 1 or zero. In this case, the metric function $f(r)$ becomes

$$f(r) = k - r^2 F(r), \tag{8}$$

and $F(r)$ still obeys the equation (5) with Ω_D replaced by Σ_D. Following Myers and Simon [9], we can also make detailed analysis for the solution (8) on its the horizon structure. Note that only when the solution is asymptotically anti-de Sitter, black hole horizon will appear for any k; when the solution is asymptotical flat, it is possible to have black hole horizon only for the case $k = 1$; when the solution is asymptotically de Sitter, a cosmological horizon appears, of course, the black hole may also occur for $k = 1$ in this case. When the solution is asymptotically de Sitter, the solution (8) with any k is the generalization of the topological de Sitter spaces introduced in [18].

For our purpose, without any detailed analysis, we just assume that a black hole horizon exists for the solution (7). Although the solution (7) looks involved, we will show that some thermodynamic quantities associated with the black hole can have simple expressions in terms of horizon radius. According to the metric (7), the black hole horizon radius r_+ is determined via the equation $f(r_+) = 0$. Due to the equation (8), one has

$$r_+^2 = k/F(r_+). \tag{9}$$

Note that when $k = 0$, it corresponds to $r_+ = 0$ or $F(r_+) = 0$. For the former case, it indicates that the black hole horizon coincides with the singularity at $r = 0$. The Hawking temperature of the black hole can be obtained by using the periodicity of imaginary time in the metric. Continuing the black hole solution to its Euclidean section via $\tau = it$, the resulting manifold will have a conical singularity at the black hole horizon r_+ if the period β of the Euclidean time τ is arbitrary. To remove the conical singularity, the period must be fixed to a special value. The periodicity of the Euclidean time appears in the quantum field's Euclidean propagator when one considers a certain quantum field in the black hole background. In quantum field theory at finite temperature, the period of the Euclidean time is explained as the inverse temperature, which is just the inverse Hawking temperature of black hole. For the black hole solution (7), the special value of the period of the Euclidean time is found to be

$$\beta \equiv 1/T = 4\pi/f'(r)|_{r=r_+}, \tag{10}$$

where a prime denotes the derivative with respect to r. That is, the Hawking temperature is

$$T = \frac{1}{4\pi} f'(r)|_{r=r_+} = -\frac{1}{4\pi}(2k/r_+ + r_+^2 F'(r)|_{r=r_+}). \tag{11}$$

Here we have used the relation (9). To get a simplified expression, we have from the equation (5) the mass of black hole in terms of the horizon radius

$$\begin{aligned} M &= \frac{(D-2)\Sigma_D r_+^{D-1}}{16\pi G} \sum_{n=0}^{m} \hat{c}_n F^n(r_+) \\ &= \frac{(D-2)\Sigma_D r_+^{D-1}}{16\pi G} \sum_{n=0}^{m} \hat{c}_n (k r_+^{-2})^n. \end{aligned} \tag{12}$$

When $k = 0$, it reduces to

$$M_{k=0} = \frac{(D-2)\Sigma_D r_+^{D-1}}{16\pi G} \hat{c}_0. \tag{13}$$

To obtain $F'(r)|_{r=r_+}$, taking derivatives on both sides of equation (5) with respect to r and using (12) and (9), we then get

$$F'(r)|_{r=r_+} = -\frac{(D-1)\sum_{n=0}^{m} \hat{c}_n (k r_+^{-2})^n}{r_+ \sum_{n=1}^{m} n \hat{c}_n (k r_+^{-2})^{n-1}}. \tag{14}$$

Substituting into (11), we reach the expression of the Hawking temperature

$$T = \frac{\sum_{n=0}^{m} (D-2n-1)\hat{c}_n k (k r_+^{-2})^{n-1}}{4\pi r_+ \sum_{n=1}^{m} n \hat{c}_n (k r_+^{-2})^{n-1}}. \tag{15}$$

When $k=0$, we find a very simple expression

$$T_{k=0} = \frac{(D-1)\hat{c}_0}{4\pi} r_+, \tag{16}$$

which is remarkable result: the Hawking temperature does not explicitly depend on other constants $\hat{c}_n (n > 1)$.

Another important thermodynamic quantity associated with black hole horizon is its entropy. Black hole behaves as a thermodynamic system, its thermodynamic quantities must obey the first law of thermodynamics, $dM = TdS$. Using this relation, in Ref. [14] we have derived the black hole entropy in a higher derivative gravity theory, and in [5] we have obtained the same entropy of Gauss-Bonnet black holes as the Euclidean approach [9]. Here we use the first law to get the entropy of black hole (7). Integrating the first law, we have

$$S = \int T^{-1} dM = \int_0^{r_+} T^{-1} \frac{\partial M}{\partial r_+} dr_+. \tag{17}$$

Here we have assumed that the entropy vanishes when the horizon radius shrinks to zero. Thus once the Hawking temperature and the mass are given in terms of the horizon radius, one can obtain the entropy of black hole using (17). Substituting (12) and (16) into (17), we arrive at

$$S = \frac{\Sigma_D r_+^{D-2}}{4G} \sum_{n=1}^{m} \frac{n(D-2)}{D-2n} \hat{c}_n (k r_+^{-2})^{n-1}. \tag{18}$$

Once again, the case with $k=0$ is very special. In that case, one can see from (18) that there is only one term with $n=1$ has the contribution to the entropy:

$$S_{k=0} = \frac{\Sigma_D r_+^{D-2}}{4G}. \tag{19}$$

Note that $\hat{c}_1 = 1$ and $\Sigma_D r_+^{D-2}$ is just the horizon area of black hole. We therefore conclude that in spite of the higher derivative terms, the entropy of black holes with $k=0$ always obeys the area formula of black hole entropy. For other cases with $k = \pm 1$, the area formula of black hole entropy does no longer hold obviously.

Some remarks are in order here. First we notice that although the asymptotic behavior and horizon structure of the black hole solution (7) are complex, their thermodynamic quantities have simple expressions in terms of horizon radius. Their mass, Hawking temperature and entropy are given by (12), (15) and (18), respectively. Second, when the horizon is a Ricci flat hypersurface, namely $k=0$, the thermodynamic quantities of the black hole have quite simple forms given by (13), (16) and (19), respectively. Further, from the relations (13) and (16) of mass and Hawking temperature to the cosmological constant $\hat{c}_0$, one has to have $\hat{c}_0 > 0$, a negative cosmological constant, in order to make

these relations sense. In addition we see from (13) and (16) that the black holes with $k = 0$ are always thermodynamically stable with positive heat capacity.

Third, as mentioned above, in spite of the asymptotic behavior of the Gauss-Bonnet black hole solution, the Gauss-Bonnet black holes with positive Gauss-Bonnet coefficient $\hat{c}_2$ always have a thermodynamically stable branch with small horizon radius in $D = 5$ dimensions, while their thermodynamic properties are qualitatively similar to the case without the Gauss-Bonnet term if $D \geq 6$. Here we show that this feature persists for gravity with higher dimensional Euler density. For example, let us consider a gravity theory consisting of a cosmological constant term $\hat{c}_0$, a curvature scalar term R and a $2n$-dimensional Euler density $\mathcal{L}_n$. From (15), we have the Hawking temperature of the black hole

$$T = \frac{(D-1)\hat{c}_0 r_+^{2n} + (D-3) r_+^{2n-2} + (D-2n-1)\hat{c}_n}{4\pi r_+ (r_+^{2n-2} + n\hat{c}_n)}, \tag{20}$$

where we have already set $k = 1$. Suppose $\hat{c}_n > 0$, which makes the horizon radius r_+ have minimal value $r_+ = 0$ (cf. [5, 8, 9]), we can easily see that the behavior of the Hawking temperature crucially depends on spacetime dimension. When $D = 2n + 1$, the Hawking temperature always increases monotonically from $T = 0$ at $r_+ = 0$ for small horizon radius, independent of the cosmological constant $\hat{c}_0$. This is consistent with the fact that at much smaller scale than the cosmological radius $1/\sqrt{|\hat{c}_0|}$, (if it does not vanishes), the cosmological constant has a negligible effect on physics on that scale. Therefore in this case the small black hole is thermodynamically stable with positive heat capacity. Of course, for larger black holes, the behavior of Hawking temperature depends on the asymptotic behavior of the black hole solutions. From (20) it can be seen that the effect of the coefficient $\hat{c}_n$ is small when $r_+^{2n-2} > n\hat{c}_n$. On the other hand, when $D > 2n+1$, the Hawking temperature always decreases monotonically from infinity at $r_+ = 0$ for small black holes, which implies that the heat capacity is negative, as the case without the Euler density term. For larger horizon radius, the effect of the Euler density term is once again small. Therefore the thermodynamic behavior of these black holes is qualitatively same as the case without the Euler density term as $D > 2n + 1$.

Finally we see from the entropy (18) that the first term is just quite familiar area term of black hole horizon, other terms comes from contributions of higher dimensional Euler densities. In this expression the cosmological constant term does not appear explicitly. This is an expected result since entropy of black hole is a function of horizon geometry [20]. Here we see that horizon topology also plays an important role for entropy of black holes in gravity with higher derivative terms. To see further the feature that black hole entropy is a character of horizon geometry and topology, let us add a Maxwell field to the Lovelock

gravity (1). We will see that entropy of the charged black hole in Lovelock gravity still have the expression (18). That is, the electric charge q does not appear explicitly in the black hole entropy expressed in terms of horizon radius. When a Maxwell field is present, we have a charged black hole solution with metric (7). Here metric function $f(r)$ is still given by (8), but $F(r)$ has to satisfy [9, 19]

$$\sum_{n=0}^{m} \hat{c}_n F^n(r) = \frac{16\pi GM}{(D-2)\Sigma_D r^{D-1}} - \frac{q^2}{r^{2D-4}}. \tag{21}$$

In this case, black hole horizon r_+ is still determined by the equation $f(r_+) = 0$. So the mass of black hole can be expressed in terms of horizon radius r_+ and charge q

$$M = \frac{(D-2)\Sigma_D r_+^{D-1}}{16\pi G}\left(\sum_{n=0}^{m} \hat{c}_n (k r_+^{-2})^n + \frac{q^2}{r_+^{2D-4}}\right). \tag{22}$$

The Hawking temperature is found to be

$$T = \frac{\sum_{n=0}^{m}(D-2n-1)k\hat{c}_n(kr_+^{-2})^{n-1} - (D-3)q^2/r_+^{2D-6}}{4\pi r_+ \sum_{n=1}^{m} n\hat{c}_n(kr_+^{-2})^{n-1}}. \tag{23}$$

The variation of the mass (22) with respect to the horizon radius r_+ is

$$\left(\frac{\partial M}{\partial r_+}\right)_q = \frac{(D-2)\Sigma_D r_+^{D-4}}{16\pi G}\left(\sum_{n=0}^{m}(D-2n-1)k\hat{c}_n(kr_+^{-2})^{n-1} - (D-3)q^2/r_+^{2D-6}\right). \tag{24}$$

Using (17) once again, and keeping q as a constant in calculation, we get

$$S = \frac{\Sigma_D r_+^{D-2}}{4G}\sum_{n=1}^{m}\frac{n(D-2)}{D-2n}\hat{c}_n(kr_+^{-2})^{n-1}. \tag{25}$$

It has a same form as the entropy (18) of a uncharged black hole.

In summary we have first generalized the static, spherically symmetric black hole solution in Lovelock gravity to the case where black hole horizon can be a positive, zero or negative constant curvature hypersurface. Although the geometry and horizon structure of the black hole solution could be quite complicated, in terms of horizon radius, we have found that some thermodynamic quantities like the black hole mass, Hawking temperature and entropy, have simple expressions. In particular, the case with Ricci flat horizon is remarkably simple: these black holes are thermodynamically stable with a positive heat capacity and their entropy always obeys the horizon area formula. By explicit calculation, it has been shown that black hole entropy depends on not only the horizon geometry, but also the horizon topology structure. In addition, the feature has been found to be universal that for black hole solutions in gravity of Ricci scalar plus a $2n$-dimensional dimensional

Euler density, when $D = 2n + 1$, the thermodynamically stable small black holes always appear with a positive heat capacity, which are absent in the case without the Euler density term. In $D > 2n + 1$, however, the thermodynamic properties of black holes become qualitatively similar to those of black holes without the Euler density term. This generalized the discussions of Gauss-Bonnet black holes to a more general case.

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References

[1] D. Lovelock, J. Math. Phys. **12**, 498 (1971).

[2] D. G. Boulware and S. Deser, Phys. Rev. Lett. **55**, 2656 (1985).

[3] B. Zwiebach, Phys. Lett. B **156**, 315 (1985).

[4] J. T. Wheeler, Nucl. Phys. B **268**, 737 (1986); Nucl. Phys. B **273**, 732 (1986).

[5] R. G. Cai, Phys. Rev. D **65**, 084014 (2002) [arXiv:hep-th/0109133].

[6] R. G. Cai and Y. S. Myung, Phys. Lett. B **559**, 60 (2003) [arXiv:hep-th/0210300].

[7] J. P. Gregory and A. Padilla, Class. Quant. Grav. **20** (2003) 4221 [arXiv:hep-th/0304250].

[8] R. G. Cai and Q. Guo, arXiv:hep-th/0311020.

[9] R. C. Myers and J. Z. Simon, Phys. Rev. D **38**, 2434 (1988).

[10] S. Nojiri, S. D. Odintsov and S. Ogushi, Phys. Rev. D **65**, 023521 (2002) [arXiv:hep-th/0108172]; M. Cvetic, S. Nojiri and S. D. Odintsov, Nucl. Phys. B **628**, 295 (2002) [arXiv:hep-th/0112045].

[11] Y. M. Cho and I. P. Neupane, Phys. Rev. D **66**, 024044 (2002) [arXiv:hep-th/0202140]; I. P. Neupane, Phys. Rev. D **67**, 061501 (2003) [arXiv:hep-th/0212092]. I. P. Neupane, arXiv:hep-th/0302132.

[12] M. Banados, C. Teitelboim and J. Zanelli, Phys. Rev. D **49**, 975 (1994) [arXiv:gr-qc/9307033].

[13] J. Crisostomo, R. Troncoso and J. Zanelli, Phys. Rev. D **62**, 084013 (2000) [arXiv:hep-th/0003271].

[14] R. G. Cai and K. S. Soh, Phys. Rev. D **59**, 044013 (1999) [arXiv:gr-qc/9808067].

[15] R. Aros, R. Troncoso and J. Zanelli, Phys. Rev. D **63**, 084015 (2001) [arXiv:hep-th/0011097].

[16] L. F. Abbott and S. Deser, Nucl. Phys. B **195**, 76 (1982); S. Deser and B. Tekin, Phys. Rev. Lett. **89**, 101101 (2002) [arXiv:hep-th/0205318]; S. Deser and B. Tekin, Phys. Rev. D **67**, 084009 (2003) [arXiv:hep-th/0212292]; A. Padilla, Class. Quant. Grav. **20**, 3129 (2003) [arXiv:gr-qc/0303082].

[17] J. P. S. Lemos, Phys. Lett. B **353**, 46 (1995) [arXiv:gr-qc/9404041]; J. P. S. Lemos and V. T. Zanchin, Phys. Rev. D **54**, 3840 (1996) [arXiv:hep-th/9511188]. C. G. Huang and C. B. Liang, Phys. Lett. A **201**, 27 (1995); R. G. Cai and Y. Z. Zhang, Phys. Rev. D **54**, 4891 (1996) [arXiv:gr-qc/9609065]; R. G. Cai, J. Y. Ji and K. S. Soh, Phys. Rev. D **57**, 6547 (1998) [arXiv:gr-qc/9708063]; S. Aminneborg, I. Bengtsson, S. Holst and P. Peldan, Class. Quant. Grav. **13**, 2707 (1996) [arXiv:gr-qc/9604005]. R. B. Mann, Class. Quant. Grav. **14**, L109 (1997) [arXiv:gr-qc/9607071]; D. R. Brill, J. Louko and P. Peldan, Phys. Rev. D **56**, 3600 (1997) [arXiv:gr-qc/9705012]; L. Vanzo, Phys. Rev. D **56**, 6475 (1997) [arXiv:gr-qc/9705004]; for recent papers, for example see, M. F. A. da Silva, A. Wang, F. M. Paiva and N. O. Santos, Phys. Rev. D **61**, 044003 (2000) [arXiv:gr-qc/9911013]; Y. m. Wu, M. F. A. da Silva, N. O. Santos and A. z. Wang, Phys. Rev. D **68**, 084012 (2003) [arXiv:gr-qc/0309002], and references therein.

[18] R. G. Cai, Y. S. Myung and Y. Z. Zhang, Phys. Rev. D **65**, 084019 (2002) [arXiv:hep-th/0110234].

[19] D. L. Wiltshire, Phys. Lett. B **169**, 36 (1986).

[20] R. M. Wald, Phys. Rev. D **48**, 3427 (1993) [arXiv:gr-qc/9307038]; T. Jacobson and R. C. Myers, Phys. Rev. Lett. **70**, 3684 (1993) [arXiv:hep-th/9305016].